Project systems theory

21/01/2016, Thursday, 14:00 – 17:00

You are **NOT** allowed to use any type of calculators

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1 (4 + 16 = 20 pts) **Linearization**

Consider the nonlinear model of a bioreactor described by the equations

$$
\begin{aligned}
p &= \mu(q)p - \frac{1}{2}p \\
q &= -\mu(q)p + \frac{2-q}{2} \\
y &= q + kq^3
\end{aligned}
$$

where $p \geqslant 0$, $q \geqslant 0$ are the states, y is the output, k is a constant, and

$$\mu(q) = 2\frac{q}{1+2q+q^2}.$$

(a) For a nonlinear system of the form $x(t) = f(x(t))$, a (constant) vector $\bar{x}$ is called an *equilibrium point* if $f(\bar{x}) = 0$ Determine the equilibrium points of the above bioreactor system.

(b) For all equilibrium points, determine the linearized model.

2 (20 pts) **Kharitonov criterion**

Suppose that a is a positive number Determine all values of a such that the interval polynomials

$$p(s) = s^3 + [a, 2a]s^2 + [2a, 3a]s + [3a, 4a]$$

are all stable

3 (5 + 20 = 25 pts) **Feedback stabilization**

Consider the system $\dot{x} = Ax + bu$ where

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

(a) Is it controllable?

(b) Find a state feedback of the form $u = k^T x$ such that the closed loop system has poles at -1 (**Hint.** You may want to use the following:

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & -2 & 1 \\ 1 & -2 & 1 & 1 \\ -2 & 1 & 1 & 1 \\ 1 & 1 & 1 & -2 \end{bmatrix}.$$

4 $(1+2+2+4+4+6+6=25$ pts) **Stabilizability and detectability**

Consider the linear system

$$x = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$$

Explain your answers to the following questions:

(a) Is it stable?

(b) Is it controllable?

(c) Is it observable?

(d) Is it stabilizable?

(e) Is it detectable?

(f) Does there exist an observer of the form $\hat{x} = P\hat{x} + Qu + Ry$?

(g) Does there exist a stabilizing dynamic compensator (from y to u)?

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10 pts free